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TITLE INVESTIGATIONS OF THE QUASI-STEADY APPROACH USED IN TRANSIENT TWO-PHASE FLOW ANALYSIS

AUTHOR(S) Kemal O. Pasamehmetoglu, N-9
Ralph A. Nelson, N-9

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Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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NOMENCLATURE

C_p	Specific heat of liquid (J/kg · K)
E	Error associated with the quasi-steady solution (dimensionless)
f, F	Notation for functional relationships
h_{fg}	Latent heat of vaporization (J/kg)
L	Height of the vapor chamber (m)
P	Pressure (Pa)
R	Droplet radius (m)
t	Time (s)
t'	Dummy variable of integration
T	Droplet mixing cup temperature (K)
T_{sat}	Saturation temperature (K)
u	Droplet velocity (m/s)
X	Independent variable
Y	Dependent variable
z	Axial coordinate (m)
z_f	Axial coordinate of the liquid front (m)
α	Liquid thermal diffusivity (m^2/s)
β_s	Steady void fraction (dimensionless)
Δt	Time-step size (s)
Δz	Distance traveled by the droplet during Δt (m)
ρ_ℓ	Liquid density (kg/m^3)
ρ_v	Vapor density (kg/m^3)
τ	Exponential period of a transient (s)
τ_c	Phenomenological time constant (s)
τ_ℓ	Droplet lifetime ($\tau_\ell = L/u$) (s)

Dimensionless Groups

$a = \Delta t/\tau$

$A = \left[4324.6P_o/(T_{sat,o} - T_o)^{4.4843}\right](\rho_\ell/\rho_{v,o})$

$B = T_o - 255.2/T_{sat,o} - T_o$

$Ja = h_{fg}/C_p(T_{sat} - T_o)$

$S = (\mathrm{d}Y_{SS}/\mathrm{d}X)/(Y_{SS}/X)$

$T^* = T - T_o/T_{sat,o} - T_o$

$T^*_{sat} = T_{sat} - T_o/T_{sat,o} - T_o$

$t^* = \alpha t/R^2$

$u^* = uR^2/\alpha L$

$z^* = z/L$

$\Delta t^* = \alpha \Delta t/R^2$

$\theta = \tau/\tau_c$

$\tau^* = \alpha\tau/R^2$

$\tau^*_\ell = \alpha\tau_\ell/R^2$

Subscripts

c	Critical value
i	Index
o	Initial
QS	Quasi-steady
SS	Steady state
TR	Transient

Superscript

j	Time counter

INVESTIGATION OF THE QUASI-STEADY APPROACH USED IN TRANSIENT TWO-PHASE FLOW ANALYSIS*

by

Kemal O. Pasamehmetoglu and Ralph A. Nelson

Safety Code Development
Nuclear Technology and Engineering Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

In this paper, implications of the quasi-steady approach to numerical solutions of two-phase flow problems are addressed by the application of basic principles. First, a simple criterion to determine the limitations of the quasi-steady approach is discussed. This criterion is used to determine the minimum time-step size required during the quasi-steady solution. Using this same concept, a method for making truly transient problems artificially quasi-steady is developed. Finally, these concepts are applied to a simple interfacial heat-transfer problem. The numerical instability that results from the quasi-steady approach during the explicit solution of this problem is investigated.

I. INTRODUCTION

The coupled-equation set that combines the single-phase multidimensional fluid conservation equations for mass, momentum, and energy with the equation for heat diffusion within a bounding wall is called the conjugate problem. If the necessary initial and boundary conditions are known, its solution requires no *a priori* knowledge of the wall-to-fluid convective heat transfer. This approach has been used to obtain both analytical and numerical solutions to many single-phase transients (see, for instance, the studies of Sucec [1,2]). Results from this approach have been compared with results obtained using a quasi-steady approach, and these comparisons have produced an understanding of when the simpler quasi-steady approach produces valid results.

The quasi-steady approach assumes a knowledge of the wall-to-fluid heat transfer based on the local-instantaneous fluid parameters. The method works as long as the fluid responds more quickly than the wall. For example, the fluid boundary layer responds so quickly that the

* This work was performed under the auspices of the US Department of Energy

surface temperature of a thick, high-conductivity wall does not have time to change. However, when the wall changes faster than the fluid, transient constitutive relations must be known to solve either problem accurately. Each transient yields unique rate-dependent constitutive relationships. Therefore, similar relationships for each phenomenon during different types of transients must be obtained before a truly transient problem can be solved.

The difficulty in solving the transient two-phase flow problem becomes even more pronounced because a third set of field equations for the additional phase must be solved simultaneously. Despite the difficulties and limitations of the quasi-steady solution approach to the conjugate problem, especially for two-phase flow, this approach is the only method available to simulate transient conditions in large, complex, two-phase systems such as chemical or nuclear power plants. In the codes developed to address these problems, the quasi-steady approach is used for both wall-to-fluid and interfacial heat transfer as well as the wall-to-fluid and interfacial-drag packages. Often, these large numerical codes are used even when the quasi-steady assumption is invalid. The literature contains many papers in which the constitutive relationships have been *improved* to obtain agreement with what is, in fact, transient data. Although valuable insight can be gained in this process, the improved constitutive relationships may be misleading and may produce inaccurate results when applied to other non-quasi-steady experiments or truly quasi-steady transients.

Therefore, when using large transient analysis codes such as the Transient Reactor Analysis Code (TRAC),[3] it is extremely important that the limitations of the quasi-steady approach are recognized. In order to obtain reliable results, the difficulties that are caused by the quasi-steady approach must be resolved. These difficulties may be classified in two categories.

1. If the time constant of the transient is smaller than the time constant of the occurring phenomena, the quasi steady approach yields erroneous results. The error increases as the transient accelerates. The numerical portion of this problem, which has long been acknowledged, has led to artificial averaging or limiting techniques, because transient constitutive relationships either do not exist or are impossible to incorporate into the quasi-steady logic of the code numerics. Often, these averaging and limiting techniques are *ad hoc* models with little experimental or theoretical support. Thus, recognizing these fast transients and developing a method to make them artificially quasi-steady are still important problems in the area of transient two-phase flow code development.

2. The second problem is associated with the time-step size used in the numerical solution of the transient two-phase field equations. As a rule of thumb, choosing a time-step size between the time constant of the phenomena and the time constant of the transient leads to smooth, valid results. However, in integral codes such a criterion may not always be satisfied because (i) the transient may be so fast that the transient time constant is smaller than the phenomenological time constant, and/or (ii) the time constants of different phenomena that are being analyzed simultaneously in different parts of the system may differ considerably such that a large enough time-step size for one phenomenon may be too small for another, and/or (iii) the choice of the time-step size may be dominated by other considerations such as the material Courant limit. If, for any reason, a time-step size smaller than the time constant of the phenomena is chosen, such a choice may cause the phenomena

to change too quickly. Especially in integral system analysis codes like TRAC,[3] such an unnatural event may be enhanced from one constitutive relation to another. Consequently, the analysis not only may yield erroneous results but also, in some cases, may create a numerical instability. This problem also has been recognized. However, a comprehensive and systematic approach to avoid this problem does not exist because, by the time the erroneous solution is obtained or an instability emerges in integral codes, the origin of the initial unnatural event(s) may be difficult to trace.

In this paper, the above difficulties of the quasi-steady approach are addressed by the application of basic principles. In Sec. II, a simple criterion to determine the limitations of the quasi-steady approach is discussed. Based upon this criterion, the time-step size requirements in the numerical solutions are discussed in Sec. III. In the same section, a systematic procedure for making transient problems artificially quasi-steady also is discussed. In Sec. IV, a simple interfacial heat-transfer problem, where the quasi-steady approach leads to a numerical instability, is investigated. Finally, the summary and conclusions are presented in Sec. V.

II. A SIMPLE CRITERION TO DETERMINE THE LIMITATION OF THE QUASI-STEADY APPROACH

A generic and systematic discussion of the quasi-steady versus transient heat-transfer problems is provided by Nelson.[4,5] This discussion is based upon the total rate of change of the dependent variable when the independent variable(s) is (are) under transient. For example, if we assume a simple steady-state constitutive relationship in the form,

$$Y_{SS} = F(X_i) \quad , \tag{1}$$

where i is the index denoting the different independent variables. If the independent variable, X_i, changes with time, then the time, t, must enter into the constitutive relationship as another independent variable as follows,

$$Y_{TR} = F(t, X_i) \quad . \tag{2}$$

As a result, the total rate of change of the dependent variable, Y_{TR}, becomes

$$\frac{dY_{TR}}{dt} = \frac{\partial Y_{TR}}{\partial t} + \sum_{i=1}^{N} \frac{\partial Y_{TR}}{\partial X_i} \frac{dX_i}{dt} \quad , \tag{3}$$

where N is the total number of time-dependent independent variables. In Eq. (3), if

$$\left| \frac{\partial Y_{TR}}{\partial t} \right| << \left| \frac{\partial Y_{TR}}{\partial X_i} \frac{dX_i}{dt} \right| \quad ,$$

and

$$\left| \frac{\partial Y_{TR}}{\partial t} \right| << \left| \sum_{i=1}^{N} \frac{\partial Y_{TR}}{\partial X_i} \frac{dX_i}{dt} \right| \quad ,$$

then the problem becomes quasi-steady for both separate- and combined-effects transients, respectively. Consequently, a steady-state constitutive relationship may be used to quantify the transient-dependent parameter, Y_{TR}, such that

$$Y_{\mathrm{TR}} \simeq Y_{\mathrm{QS}} = F\{X_i(t)\} \quad . \tag{4}$$

Further discussion of Eqs. (2), (3), and (4) may be found in the studies of Nelson.[4,5] Equation (3), which has merit because of its original discussion of the quasi-steady versus transient problems, provides a sound mathematical basis for differentiating them. However, the practical use of this equation is difficult. The different terms on the right-hand side (RHS) of Eq. (3) cannot be quantified easily. Because the determination of whether a problem is quasi-steady or not is based upon the relative magnitude of these terms, Eq. (3) does not lead directly to a firm criterion.

Consequently, we have tried to find a more practical equivalent to Eq. (3) that can be quantified more easily. We accomplished this by considering Eq. (3) relative to a simple generic transient model. The physical model with a single independent variable ($N = 1$) consists of a signal source that emits signals with a time-dependent property, a filter or amplifier that processes this signal in a predescribed form, and a receiver that receives the altered signals delayed by τ_c. In this simple example, τ_c may be regarded as the time required for a signal to travel from the source to the receiver. In a more general case, τ_c, which represents the time constant of the phenomena, is not necessarily constant. It may be a function of the characteristic properties of the signal and/or the signal processor. This model may symbolize a more concrete example for a transient heat-transfer problem in which the signal emitted may represent a time-dependent wall temperature, the processor may represent the convective heat-transfer phenomena, and the received signal may represent the fluid temperature.

Based on this simple model, the signal received at time t is equal to the delayed signal emitted at time $t - \tau_c$ and processed through the filter. Thus, if we assume that Y_{SS} is the filter (we will determine the requirements for this assumption to be valid),

$$Y_{\mathrm{TR}}(t, X) = Y_{\mathrm{SS}}\left[X(t - \tau_c)\right] \quad . \tag{5}$$

If the Taylor series expansion for small τ_c is used, the RHS of Eq. (5) can be rewritten to yield

$$Y_{\mathrm{TR}}(t, X) = Y_{\mathrm{SS}}(X) - \tau_c \frac{\mathrm{d}Y_{\mathrm{SS}}}{\mathrm{d}X}\frac{\mathrm{d}X}{\mathrm{d}t} \quad , \tag{6}$$

where higher order terms are neglected. If a parameter S is defined as

$$S = \frac{\mathrm{d}Y_{\mathrm{SS}}}{\mathrm{d}X} \Big/ \frac{Y_{\mathrm{SS}}}{X} \quad , \tag{7}$$

then Eq. (6) may be written as

$$Y_{\mathrm{TR}}(t, X) = Y_{\mathrm{SS}}\left[X(t)\right]\left[1 - S\frac{\tau_c}{X}\frac{\mathrm{d}X}{\mathrm{d}t}\right] \quad . \tag{8}$$

Equation (8) suggests that, for the quasi-steady approach to be valid, the following condition must be satisfied,

$$\left|\frac{dX}{dt}\frac{\tau_c}{X}\right| << \left|\frac{1}{S}\right| \quad . \tag{9}$$

where S can be calculated easily by using the definition given by Eq. (7), after the steady-state constitutive relationship, Y_{SS}, is known. When the inequality in Eq. (9) is satisfied, the problem is quasi-steady and

$$Y_{QS}(t, X) \simeq Y_{SS}[X(t)] \quad . \tag{10}$$

Otherwise, the problem is a true transient. In this case, Eq. (10) is no longer valid and a transient constitutive relationship is required.

It is important to note that Eq. (8) is merely an approximation for a transient constitutive relationship obtained simply by translating the steady-state constitutive relationship along the time axis by an amount τ_c. It is derived for the purpose of obtaining a criterion for the limitation of the quasi-steady approach. In reality, τ_c is not constant as treated so far in this paper. It is a function of time and the magnitudes and time rates of changes of the dependent and independent variables. Thus, each transient yields a unique constitutive relationship. However, if τ_c can be appropriately correlated as a function of these variables, then Eq. (8) may be used as a generic form for transient constitutive relationships. Equation (8) is a practical alternative to Eq. (3) because it can be used more easily by identifying and quantifying the time constants of the different phenomena.

Another commonly used qualitative criterion for the quasi-steady approach is defined in terms of the time-constants ratio. If the time constant of the transient is much greater than the time constant of the representative phenomenon, then the problem is quasi-steady. Note that, when applied to an exponential transient in the form

$$X = X_o \exp\left(\frac{-t}{\tau}\right) \quad , \tag{11}$$

Eq. (9) reduces to

$$\left|\frac{1}{\theta}\right| << \left|\frac{1}{S}\right| \quad , \tag{12}$$

where $\theta = \tau/\tau_c$. This equation readily illustrates the concept of the time-constants ratio mentioned earlier, but note that Eq. (9) is not restricted to exponential transients. For an exponential decay, we can classify the transient problem as follows,

$$\begin{aligned} &\theta >> |S| \quad \text{(quasi-steady)} \quad , \\ &\theta << |S| \quad \text{(truly transient)} \quad , \text{or} \\ &\theta \sim \ |S| \quad \text{(transition)} \quad . \end{aligned}$$

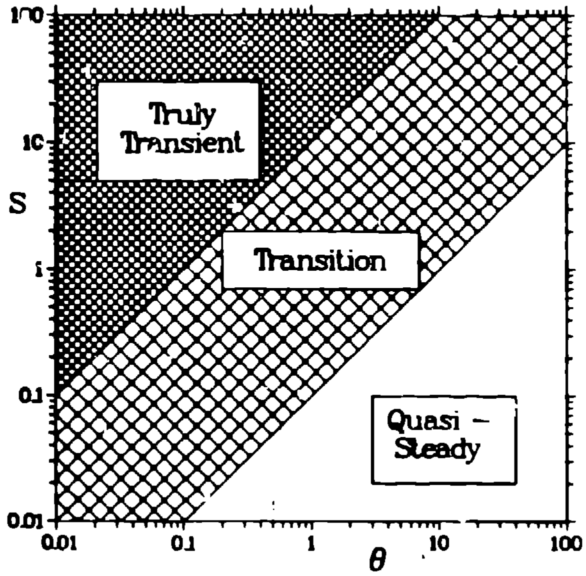

Fig. 1.
Tentative map for transient problems.

Figure 1 shows a tentative map for the quasi-steady criterion. In this figure, the boundaries between the different transients are tentatively assigned, with the assumption that $<<$ or $>>$ means an order of magnitude difference. Specifically, the difference between the transitional and the truly transient problem is not very clear but, at some point within the transition region, the quasi-steady approach becomes invalid.

The dimensionless group on the left-hand side (LHS) of Eq. (9) was discussed by Pasamehmetoglu[6] and Gunnerson[7] in the context of transient critical heat flux (CHF). The final transient CHF correlation explicitly includes the time-constants ratio in Eq. (12).[8] From an overview of Kuznetsov,[9] it appears that a similar dimensionless group is being used in the Soviet literature in conjunction with *unsteady problems*.* However, to the best of our knowledge, the English literature does not contain any information regarding the origin or the quantitative application of this dimensionless group.

The current paper is concerned with the implications of Eqs. (9) and (12) on the numerical solutions. The quasi-steady criterion is not discussed in detail here. However, it is discussed from a numerical standpoint in the next section.

III. MINIMUM TIME-STEP SIZE REQUIREMENT FOR THE QUASI-STEADY SOLUTIONS

Many real-life transients, especially in reactor safety, are such that the transient parameter rapidly increases or decreases in the early stages of the transient, then the rate of change slows. For the purpose of this paper, we assume that the transient is decreasing monotonically and

* In Soviet literature, the term *unsteady problem* is equivalent to what we refer to as a truly transient problem.

may be approximated by an exponential decrease given by Eq. (11). During the numerical solution of this type of transient, the quasi-steady solution criterion given by Eq. (9) yields

$$\frac{\tau_c \left[|X_o e^{-(t+\Delta t)/\tau} - X_o e^{-t/\tau}| \right]}{\Delta t X_o e^{-t/\tau}} << \left| \frac{1}{S} \right| \ , \tag{13}$$

where, within a time-step size Δt,

$$S = \left\{ \frac{Y\left[X(t+\Delta t)\right] - Y\left[X(t)\right]}{X(t+\Delta t) - X(t)} \right\} \left\{ \frac{X(t)}{Y\left[X(t)\right]} \right\} \ . \tag{14}$$

Rearranging Eq. (13) and defining the variable

$$a = \frac{\Delta t}{\tau} \ ,$$

we obtain

$$\frac{1 - e^{-a}}{a} << \left| \frac{\theta}{S} \right| \ . \tag{15}$$

If we assume that the system can absorb some error, E, without amplifying the error and/or without becoming unstable, the $<<$ sign may be changed to $\leq$ sign and Eq. (15) becomes

$$\frac{1 - e^{-a}}{a} \leq E \frac{\theta}{S} \ . \tag{16}$$

Because a will always be a real, positive number for cases of interest, the quantity $(1 - e^{-a})/a$ will have a maximum value of 1 at $a = 0$ and a minimum value approaching 0 as a approaches ∞ so that

$$0 < \frac{1 - e^{-a}}{a} \leq 1 \ .$$

The solution of Eq. (16) yields

$$a \geq f\left(\frac{E\theta}{S}\right) \ . \tag{17}$$

The solution domain for a as a function of $E\theta/S$ is shown in Fig. 2. Remember that, if $E\theta/S \geq 1$, the problem becomes quasi-steady, as discussed in Sec. II. As shown in Fig. 2, for fast transients with small θ, for systems with small error margin E, and for a quasi-steady constitutive relationship showing a strong dependence on the independent variable ($S > 1$), the required time-step size must be considerably greater than the transient time constant. For opposing trends, the ratio a decreases. This decrease, however, may be caused by a decrease in the time-step size as well as by an increase in the transient time constant τ, as the transient becomes slower. Thus, the map in Fig. 2 is not a good measure for the time-step size. A

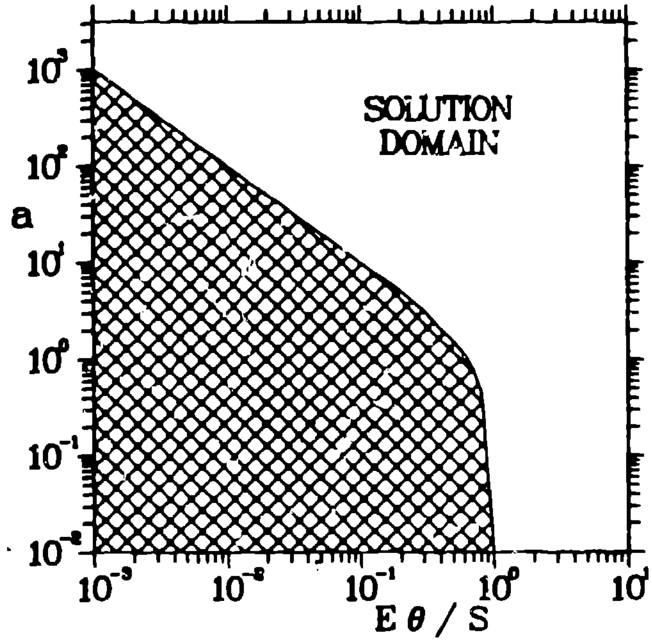

Fig. 2.
Solution domain for a.

better way of mapping the time-step size may be against the phenomenological time constant, τ_c. Manipulation of Eq. (17) yields

$$\frac{E}{S}\frac{\Delta t}{\tau_c} \geq \frac{E\theta}{S} j\left(\frac{E\theta}{S}\right) \quad . \tag{18}$$

Figure 3 shows the ratio $E\Delta t/S\tau_c$ versus $E\theta/S$. As shown in this figure, for fast transients ($E\theta/S < 0.2$), the dimensionless group $E\Delta t_c/S\tau_c$ becomes a constant and equal to 1. For different error margins, the critical time-step size, Δt_c, may be calculated as

$$\Delta t_c = \frac{1}{E} S\tau_c \quad . \tag{19}$$

Thus, if a time-step size larger than the critical time-step size is chosen, the numerical solution will proceed without allowing the dependent variable Y to change too rapidly. However, for certain phenomena with large time constants, this critical time-step size may be quite large, as shown in Fig. 3. We may not want to use the numerical scheme in such large time steps because it may either introduce numerical errors and/or mask what is happening within that large a time step. Often, such a large time-step size also may contradict other

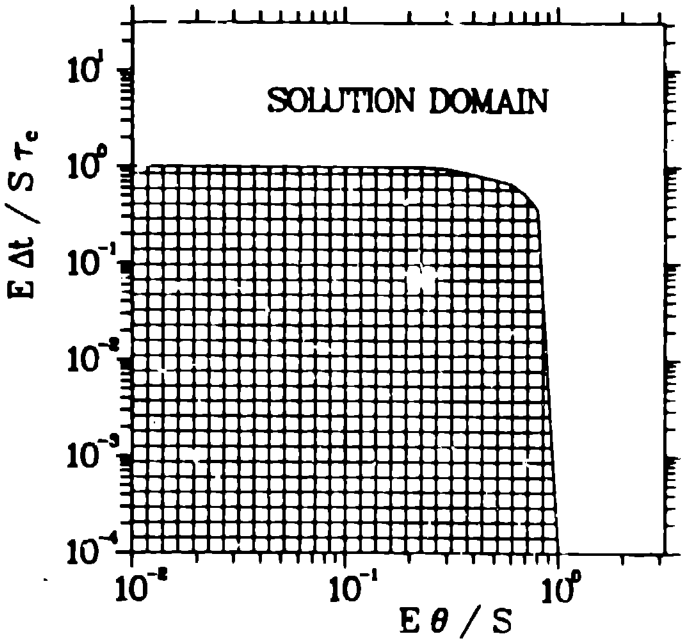

Fig. 3.
Solution domain for $\Delta t/\tau_c$.

considerations such as the material Courant limit. Next, we consider what happens if the numerical solution is forced to use time-step sizes smaller than the critical time-step size.

The upper portion of Fig. 4 shows an exponential transient where the independent variable X decreases with time according to Eq. (11). The quasi-steady variation of the dependent variable Y with respect to X also is shown in the lower portion of the figure. For this example, a linear relationship is used. If the time-step size is equal to the critical time-step size, X is assumed to change from point 1 to point 4 along the dotted line 1–4, with a slope that satisfies the quasi-steady criterion. Thus, at point 4, the error introduced by the quasi-steady approach is limited to E.* However, if a time-step size smaller than Δt_c is chosen, then at intermediate points 2 and 3, the criterion for the quasi-steady approach is violated and an error greater than E results. To avoid such errors in the prediction of Y that potentially may lead to a numerical instability, we can use an artificial correlation $Y(X)$ that follows the dotted line 1–2′-3′–4. The point $Y_{2'}$ is calculated as $Y(X_{2'})$. In the $X-t$ plane, $X_{2'}$, which is located along the dotted line 1–4, corresponds to the quasi-steady equivalent of X_2 at time $t = t_2$, as shown in Fig. 4. Therefore, at time t_2, the quasi-steady correlation for Y is evaluated based upon the value of $X_{2'}$ instead of the value of X_2. The value of $Y_{3'}$ is calculated through an identical method. This procedure results in a slower change in Y within

* As the time-step size becomes larger, the difference equations deviate from the original differential equations and produce larger numerical errors. However, such numerical errors are not considered in this study. Within the context of this paper, the error E refers to the error that occurs because of the quasi steady approach.

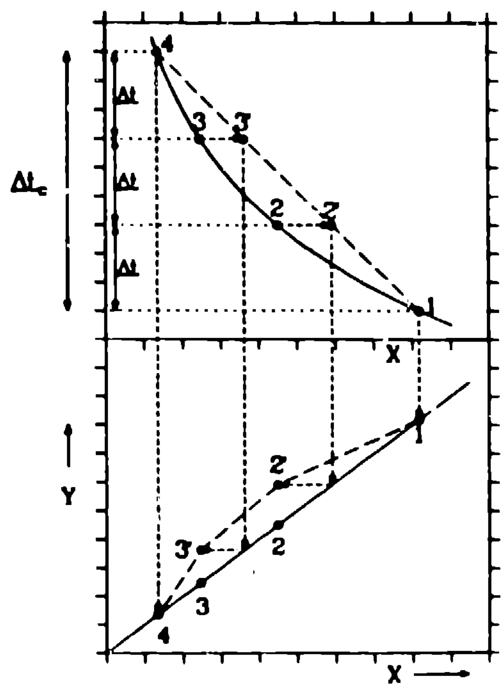

Fig. 4.
Illustration of the minimum time-step size and artificial quasi-steady method for exponentially decreasing transients.

the critical time-step size until X reaches point 4; thus, the unrealistically rapid changes that may occur early in the evaluation are avoided.

We refer to the procedure shown in Fig. 4 as the artificial quasi-steady approach. How well this method approximates a truly transient constitutive relationship can be determined only when transient constitutive relationships become available. Nevertheless, the values of Y calculated through this approach are more realistic than those obtained from the plain quasi-steady approach. Thus, this method does not introduce unrealistic changes in Y that may produce errors with high orders of magnitudes and, sometimes, numerical instabilities.

So far our discussion has concentrated on the exponentially decreasing transients. Similar arguments are valid for other types of transients that produce transient curves that become level as time increases. However, other transients that do not show a tendency to become level after a certain time may require a more complicated analysis. One such example would be a linear increase of X with respect to time, as shown in Fig. 5. If the slope of the line that represents X versus t is greater than the slope dictated by the quasi-steady criterion given by Eq. (9), the problem becomes truly transient. In this case, the critical time-step size would be ∞, which means that any finite time-step size violates the quasi-steady criterion. In this case, the same procedure discussed earlier may be used to avoid rapid changes in Y. First, from Eq. (9), the line with the maximum slope dX/dt that satisfies the quasi-steady criterion may be found. This is the dotted line in Fig. 5. Thus, whereas the independent variable X changes along the transient line 1–4, the dependent variable Y is calculated using the quasi-steady equivalents of X along the line 1–4′ as shown in Fig. 5. Again, this is an artificial

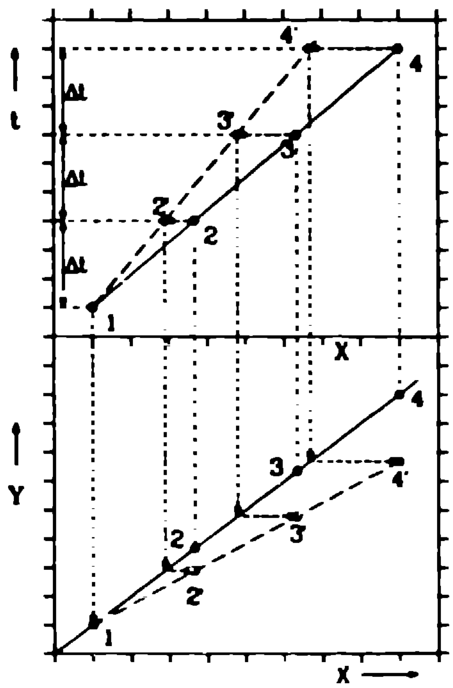

Fig. 5.
Illustration of the artificial quasi-steady method for linear transients.

quasi-steady procedure and its absolute accuracy has not been determined. Nevertheless, we claim that such an approach is an improvement over the plain quasi-steady approach and can lead to a smooth solution

In this section, we have dealt with the time-step size requirements of the quasi-steady approach. Even though violation of this requirement is expected to give erroneous results such results may not necessarily produce a numerical instability. We believe the numerical instability is a product of a *chain reaction* in systems where a number of coupled differential equations are involved. A simple coupled, two-phase flow system is discussed in Sec. IV. The numerical instability that arises from the quasi-steady solution of this system also is discussed

IV. NUMERICAL INSTABILITY CAUSED BY THE QUASI-STEADY APPROACH

To illustrate the numerical instability caused by the quasi-steady approach, we chose a simple, two-phase flow model that consists of subcooled liquid droplets injected into a saturated steam volume. Figure 6 shows a schematic description of the physical configuration. This problem is explained more thoroughly in our earlier study.[10] Our mathematical model is based on the following simplifying assumptions.

1. The amount of noncondensables in the steam is negligible
2. Droplet break-up and agglomeration are not modeled. We assume that all the injected droplets are spherical with the same radius
3. A pure conduction model for the droplets is used to estimate the condensation rates.[11,12]

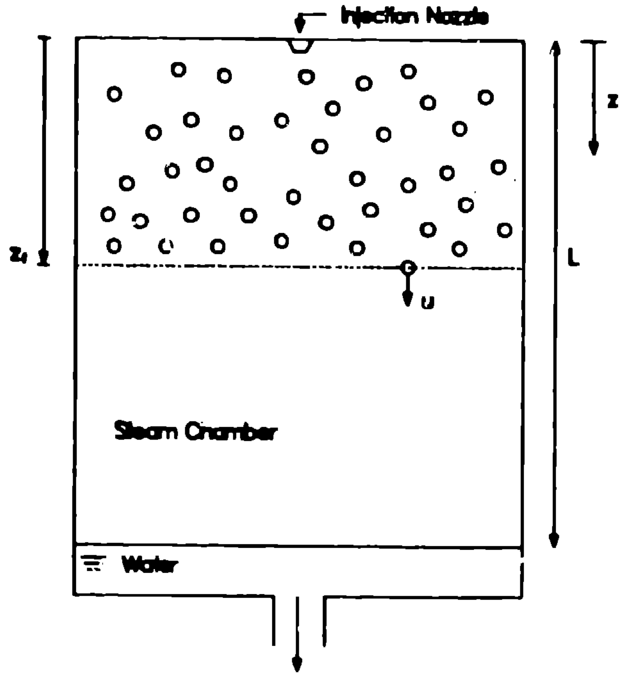

Fig. 6.
Schematic description of the interfacial condensation problem.

4. The effect[13] of the condensate-layer thermal resistance is neglected.
5. The liquid injection rate is constant. The injected liquid breaks into a homogeneous distribution of droplets that move with a constant and equal velocity.
6. The increase in droplet radius caused by condensation[11] is small and may be neglected in computing the chamber void fraction.
7. The heat transfer between the two-phase flow and chamber walls is neglected.
8. The steam-side heat transfer and the sensible heat of steam are neglected when compared with the latent heat.
9. The steam within the chamber is quasi-stationary and the inlet and exit flow rates are negligible.
10. The steam is modeled as an ideal gas.
11. The injection rates are restricted such that the resulting chamber void fraction is greater than 0.90. Therefore, the pressurization caused by the liquid injection is neglected.

For brevity, the derivation of the governing equations is not discussed here but is described in our earlier paper.[10] The chamber depressurization must be analyzed over two different time domains. The first is the early injection period that lasts until the first injected droplets reach the bottom of the chamber. This period is defined over the time interval $0 \le t^* \le \tau_\ell^*$, where τ_ℓ^* is the dimensionless droplet lifetime. During this period, the rate of change of the saturation temperature may be calculated from

$$\frac{dT_{sat}^*}{dt^*} = -\frac{A}{Ja}\left[\frac{(1-\beta_e)\frac{t^*}{\tau_\ell^*}}{1-(1-\beta_e)\frac{t^*}{\tau_\ell^*}}\right](T_{sat}^* + B)^{-3.4843}$$

$$\times\left[\frac{1}{z_f^*}\int_0^{z_f^*}\left(\frac{\partial T^*}{\partial t^*}\right)_{z^*=cst} dz^* + \frac{1}{t^*}\,T^*(t^*, z_f^*)\right] \quad , \tag{20}$$

where the dimensionless variables are defined in the nomenclature.

The second domain, the steady void-fraction period, is defined by $t^* \ge \tau_\ell^*$. During this period, the saturation temperature changes according to

$$\frac{dT_{sat}^*}{dt^*} = -\frac{A}{Ja}\left(\frac{1-\beta_e}{\beta_e}\right)(T_{sat}^* + B)^{-3.4843}\left[\int_0^1\left(\frac{\partial T^*}{\partial t^*}\right)_{z^*=cst} dz^* + \frac{1}{\tau_\ell^*}\,T^*(t^*, 1)\right] \quad . \tag{21}$$

In Eqs (20) and (21), T^* is the droplet mixing cup temperature[14] and is given by

$$T^*(t^*, z^*) = T_{SS}^*(t^*, z^*/u^*) + \frac{1}{T_{sat}^*(t^* - \frac{z^*}{u^*})}\int_0^{z^*/u^*}\frac{dT_{sat}^*}{dt'}\,T_{SS}^*\left(t^*, \frac{z^*}{u^*} - t'\right)dt' \quad , \tag{22}$$

where dT_{sat}^*/dt' is evaluated at $t^* + t' - z^*/u^*$ and T_{SS}^* is the droplet temperature[14] with the steady saturation temperature given by

$$T_{SS}^*(t^*, \frac{z^*}{u^*}) = T_{sat}^*(t^* - \frac{z^*}{u^*})\left[1 - \exp\left(-\pi^2\,\frac{z^*}{u^*}\right)\right]^{1/2} \quad . \tag{23}$$

Equation (22) represents the exact transient solution for the droplet temperature. It forms a coupled set with either Eq (20) or Eq (21) that must be solved simultaneously. If the saturation temperature changes slowly Eq (22) may be approximated by its quasi steady equivalent[14] given by

$$T^*(t^*, z^*) = T_{sat}^*(t^*)\left[1 - \exp\left(-\pi^2\,\frac{z^*}{u^*}\right)\right]^{1/2} \quad . \tag{24}$$

By substituting Eq. (24) into Eqs. (20) and (21), we obtain:

for $0 \le t^* \le \tau_\ell^*$,

$$\frac{dT^*_{sat}}{dt^*} = -\frac{A}{Ja}\left[\frac{(1-\beta_s)\frac{t^*}{\tau_\ell^*}}{1-(1-\beta_s)\frac{t^*}{\tau_\ell^*}}\right](T^*_{sat}+B)^{-3.4843}$$

$$\times\left\{\frac{1}{t^*}\,T^*_{sat}\left[1-\exp\left(-\pi^2 t^*\right)\right]^{1/2}+\frac{1}{t^*}\frac{dT^*_{sat}}{dt^*}\int\limits_0^{t^*}\left[1-\exp\left(-\pi^2 t'\right)\right]^{1/2}dt'\right\} \quad ; \qquad (25)$$

and, for $t^* \ge \tau_\ell^*$,

$$\frac{dT^*_{sat}}{dt^*} = -\frac{A}{Ja}\left(\frac{1-\beta_s}{\beta_s}\right)(T^*_{sat}+B)^{-3.4843}$$

$$\times\left\{\frac{1}{\tau_\ell^*}\frac{dT^*_{sat}}{dt^*}\frac{1}{\tau_\ell^*}\,T^*_{sat}\left[1-\exp\left(-\pi^2\tau_\ell^*\right)\right]^{1/2}+\int\limits_0^{\tau_\ell^*}\left[1-\exp\left(-\pi^2 t'\right)\right]^{1/2}dt'\right\} \quad . \qquad (26)$$

For $0 \le t^* \le 0.5$, the integrals[14] on the RHS of Eqs. (25) and (26) may be approximated by

$$\int\limits_0^{t^*}\left[1-\exp\left(-\pi^2 t'\right)\right]^{1/2}dt' = 0.85t^*\left[1-\exp\left(-\pi^2 t^*\right)\right]^{0.7} \quad . \qquad (27)$$

Thus, by substituting Eq. (27) into Eqs. (25) and (26), we obtain:

for $0 \le t^* \le \tau_\ell^*$,

$$\frac{dT^*_{sat}}{dt^*} = -\frac{A}{Ja}\left[\frac{(1-\beta_s)\frac{t^*}{\tau_\ell^*}}{1-(1-\beta_s)\frac{t^*}{\tau_\ell^*}}\right](T^*_{sat}+B)^{-3.4843}$$

$$\times\left\{0.85\frac{dT^*_{sat}}{dt^*}\left[1-\exp\left(-\pi^2 t^*\right)\right]^{0.7}+\frac{1}{t^*}\,T^*_{sat}\left[1-\exp\left(-\pi^2 t^*\right)\right]^{1/2}\right\} \quad ; \qquad (28)$$

and, for $t^* \ge \tau_\ell^*$,

$$\frac{dT^*_{sat}}{dt^*} = -\frac{A}{Ja}\left(\frac{1-\beta_s}{\beta_s}\right)(T^*_{sat}+B)^{-3.4843}$$

$$\times\left\{0.85\frac{dT^*_{sat}}{dt^*}\left[1-\exp\left(-\pi^2\tau_\ell^*\right)\right]^{0.7}+\frac{1}{\tau_\ell^*}\,T^*_{sat}\left[1-\exp\left(-\pi^2\tau_\ell^*\right)\right]^{1/2}\right\} \quad . \qquad (29)$$

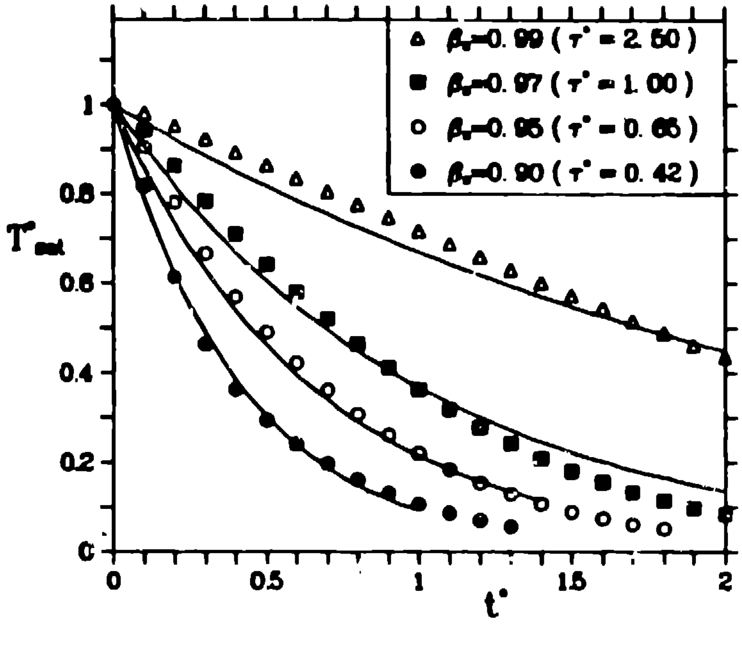

Fig. 7.
Semi-explicit solution for different injection rates.

If the terms containing dT^*_{sat}/dt^* on the LHS are combined, a semi-explicit solution is obtained for Eqs. (28) and (29). Such solutions for four different injection rates are shown in Fig. 7. This figure and all the other sample solutions reported in this paper correspond to an initial steam pressure of 1 MPa. As shown in Fig. 7, the rate of change of the saturation temperature for the cases shown may be approximated by an exponential decrease with a period reported in the figure. The semi-explicit solution does not lead to a numerical instability because the rate of change of the saturation temperature appears only on one side of the governing equation.

Next, we consider the completely explicit solution by writing the forward-marching finite-difference equations corresponding to Eqs. (28) and (29). If the * notation is eliminated, these equations can be written as follows:

for $0 \leq t \leq \tau_\ell$,

$$\frac{T^{j+1}_{sat} - T^{j}_{sat}}{\Delta t} = -\frac{A}{Ja}\left[\frac{(1-\beta_s)\frac{t}{\tau_\ell}}{1-(1-\beta_s)\frac{t}{\tau_\ell}}\right](T^{j}_{sat} + B)^{-3.4843}$$

$$\times \left\{0.85\,\frac{T^{j}_{sat} - T^{j-1}_{sat}}{\Delta t}\left[1-\exp\left(-\pi^2 t\right)\right]^{0.7} + \frac{1}{t}\,T^{j}_{sat}\left[1-\exp\left(-\pi^2 t\right)\right]^{1/2}\right\} \quad ; \qquad (30)$$

and, for $t \geq \tau_\ell$,

$$\frac{T_{sat}^{j+1} - T_{sat}^{j}}{\Delta t} = -\frac{A}{Ja}\left(\frac{1-\beta_e}{\beta_e}\right)(T_{sat}^{j} + B)^{-3.4843}$$

$$\times \left\{0.85\ \frac{T_{sat}^{j} - T_{sat}^{j-1}}{\Delta t}\left[1 - \exp\left(-\pi^2\tau_\ell\right)\right]^{0.7} + \frac{1}{\tau_\ell}\ T_{sat}^{j}\left[1 - \exp\left(-\pi^2\tau_\ell\right)\right]^{1/2}\right\} \quad . \tag{31}$$

The results of this explicit difference scheme are shown in Figs. 8 and 9 for liquid injection rates that correspond to $\beta_e = 0.9$ and $\beta_e = 0.95$, respectively. The semi-explicit solution in Fig. 7 shows that, for $\beta_e = 0.9$, the saturation temperature decreases exponentially with a period $\tau^* \simeq 0.4$. If we assume that the time constant of the phenomena is the droplet lifetime τ_ℓ^* that, in this case, is equal to 0.5, the time-constants ratio, θ, is 0.8. For this problem, S is defined as

$$S = \frac{dT^*}{dT_{sat}^*} \Big/ \frac{T^*}{T_{sat}^*} \quad ,$$

which yields 1 using Eq. (24). Thus, the problem is closer to the truly transient end of the transition region as shown in Fig. 1, and there is a minimum time-step size requirement, as discussed in Sec. III. Stable solutions require a time-step size greater than τ_ℓ^* that becomes impractical for this problem. As shown in Fig. 8, time-step sizes smaller than τ_ℓ^* lead to unstable solutions. Similar arguments are valid for Fig. 9, which shows that increasing the time-step size delays the instability. This instability is directly related to the speed of the transient because, for slower transients ($\beta_e = 0.99$), no such instability was detected, even with very small time-step sizes.

The cause of these instabilities observed in Figs. 8 and 9 may be analyzed by considering the terms within the braces in Eqs. (28) and (29). The physics of the problem requires the dimensionless saturation temperature to decrease monotonically and to approach zero asymptotically. Therefore, the term within the braces in Eq. (29) must be greater than zero. Thus,

$$0.85\left|\frac{dT_{sat}^*}{dt^*}\right|\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{0.7} < \frac{1}{\tau_\ell^*}\ T_{sat}^*\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{0.5} \quad , \tag{32}$$

which yields

$$\frac{\tau_\ell^*\left|\dfrac{dT_{sat}^*}{dt^*}\right|}{T_{sat}^*} < 1.18\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{-0.2} \quad . \tag{33}$$

Notice that Eq. (33) is analogous to the quasi steady criterion given by Eq. (9). For an exponential decrease in the saturation temperature, Eq. (33) becomes

$$\frac{\tau_\ell^*}{\tau^*} < 1.18\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{-0.2} \quad . \tag{34}$$

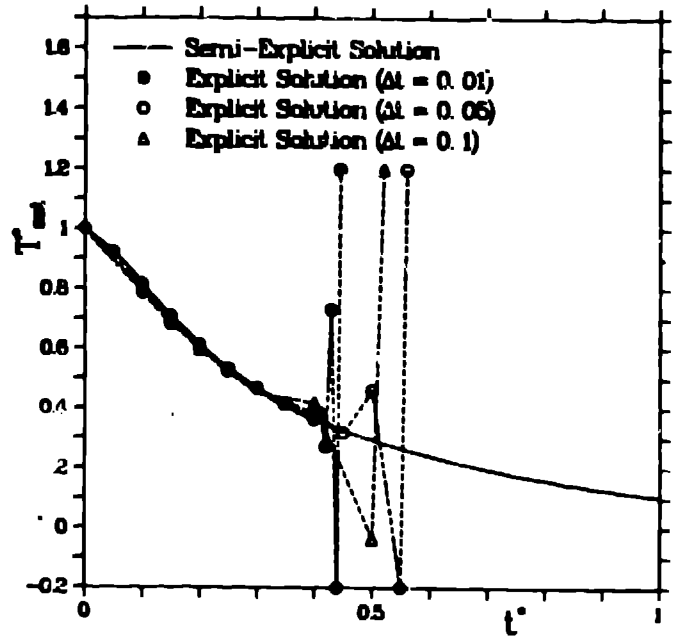

Fig. 8.
Explicit solution with different time-step sizes for $\beta_u = 0.90$ and $\tau_\ell^* = 0.5$.

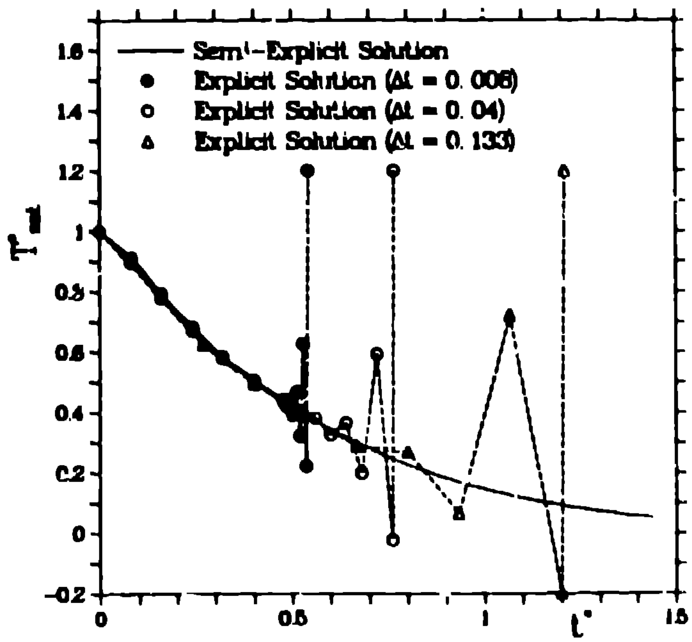

Fig. 9.
Explicit solution with different time-step sizes for $\beta_s = 0.95$ and $\tau_\ell^* = 0.4$.

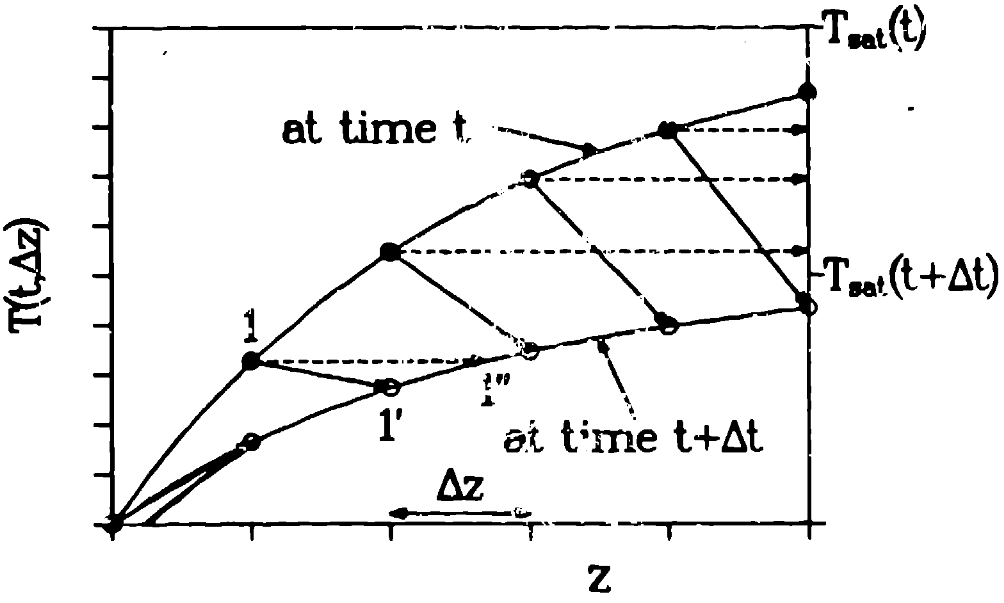

Fig 10.
Schematic illustration for the physical cause of the numerical instability.

The numerical equivalent of Eq. (33) becomes

$$\frac{\tau_\ell^*(T_{sat}^{j-1} - T_{sat}^j)}{\Delta t T_{sat}^j} < 1.16\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{-0.2} \quad , \tag{35}$$

which can be solved for the critical time-step size to yield

$$\Delta t > \Delta t_c = 0.85\,\frac{\tau_\ell^*(T_{sat}^{j-1} - T_{sat}^j)}{T_{sat}^j}\left[1 - \exp\left(-\pi^2\tau_\ell^*\right)\right]^{0.2} \quad . \tag{36}$$

Equation 36 is based on the chamber-averaged condensation rate. When the condition given by Eq. (36) is violated, the unnatural behavior of the individual droplets that results is illustrated in Fig. 10 for some location within the volume. In this figure, Δz represents the distance traveled by each droplet during the time interval Δz. Thus, $\Delta z = u\Delta t$. The change in saturation temperature within a time-step size, Δt, is shown on the right abscissa. Because of the quasi-steady assumption, the liquid temperature profile within the chamber at time t adjusts itself instantaneously to the new saturation temperature at time $t + \Delta t$.

Evaporation of the droplets towards the bottom of the chamber is possible if the depressurization is sufficiently high. However, the evaporation cannot exceed the condensation within the total volume for the given time step. The evaporating droplets must be confined within some lower portion of the chamber so that condensation exceeds evaporation for the total volume. The opposite condition suggests a net energy gain by the chamber, which means that energy can be extracted from the cold liquid into warmer vapor. This obviously violates the second law of thermodynamics. Figure 9 shows a situation where the droplet at position 1 has a lower temperature as it travels through Δz to position 1'. Therefore, the temperature of all the other droplets downstream from point 1 also decreases and a net evaporation within the chamber is produced; thus, the second law of thermodynamics is violated. Equation 36 indicates the minimum time-step size that allows a stable solution. This minimum time-step size indicates a minimum distance traveled, Δz_c. Because Eq. (36) applies to the chamber-averaged condensation rate, the corresponding Δz_c may allow certain droplets to evaporate while others yield condensation. However, the net effect will always be in favor of condensation. When analyzed for individual droplets, an overly protective stable solution can be obtained if all the droplets are forced to yield condensation by the correct choice of Δz. For example, Δz can be chosen such that the droplet in position 1 is forced to go to 1'' or farther, as shown in Fig. 10. The same restrictions apply to the other droplets shown in Fig. 10. Such an [illegible]roach obviously requires different critical space increments, Δz_c, for droplets at differer[illegible]. The critical space increment, which is suggested by Eq. (36) and based on the [illegible]-averaged condensation, provides a value between the minimum and maximum values of [illegible] computed for different droplets. Thus, certain droplets are allowed to evaporate while the solution remains stable.

Now that the criterion for a stable solution has been developed, we can make the solution artificially quasi-steady, as discussed in Sec. III. If the time-step size that results from Eq. (36) is impractically large or expected to produce high numerical errors, we can use the artificial quasi-steady approach with smaller time-step sizes. For a given time-step size, the maximum allowable value of $T^j_{sat} - T^{j-1}_{sat}$ can be calculated from Eq. (35). This value may be used on the RHS of Eqs. (30) and (31) to obtain a stable solution. Such an approach is applied to the problem when the injection rate corresponds to $\beta_s = 0.9$ and the dimensionless droplet lifetime is equal to 0.5. For a time step-size equal to 0.025, the explicit solution of this problem is unstable, as shown in Fig. 8. By using the above method to make the problem artificially quasi-steady, the solution becomes stable and the results are very close to the semi-explicit solution, as shown in Fig. 11.

V. SUMMARY AND CONCLUSIONS

In this paper, the quasi-steady approach, commonly used in transient two-phase flow problems, is investigated from the use of basic principles. In many cases, it is difficult to estimate whether a given problem is truly transient or quasi-steady. A simple criterion to detect the truly transient problems is given in Sec. II. Based on this criterion, the minimum time-step size required during numerical solutions is determined in Sec. III. Some truly transient problems may be made artificially quasi-steady and, thus, a viable numerical solution without unnaturally fast changes in the dependent variable is determined. This concept of artificially quasi-steady analysis also is discussed in Sec. III. Finally, in Sec. IV, a simple interfacial heat-transfer problem that illustrates these concepts is discussed. The problem consists of cold liquid droplets injected into a steam chamber. The governing equations are solved numerically for the rate-of-depressurization within the chamber. The stable semi-explicit solution is compared with the explicit solution that, for high injection rates and small time-step sizes, becomes unstable. The origin and results of this instability are discussed in detail. The results obtained by artificially stabilizing the problem also are reported in Sec. IV and are in good agreement with the semi-explicit solution.

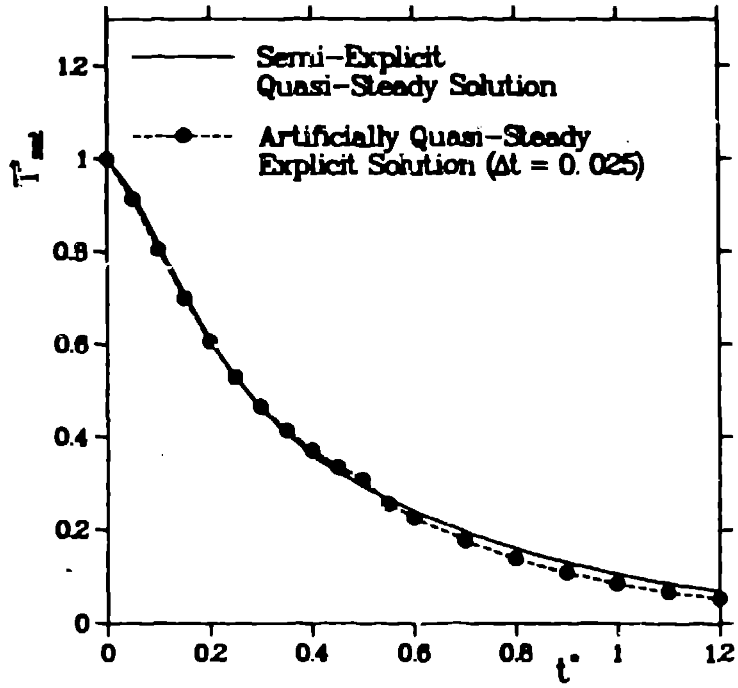

Fig. 11.
Artificially quasi-steady explicit solution for $\beta_s = 0.90$ and $\tau_\ell^* = 0.5$.

The best solution to a transient two-phase problem can be achieved by using transient constitutive relationships. However, such relationships do not exist for most possible transients and the ones that do exist are almost impossible to incorporate into the quasi-steady logic of existing computer codes. From this perspective, we believe that the concepts presented within this paper may be useful for future two-phase flow code development and assessment efforts. We recognize that, at this point, the incorporation of these concepts into integral codes is not trivial and further investigation of this subject is needed.

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